Shepherd University
College of Science, Technology, Engineering, and Mathematics
P.O. Box 5000
Shepherdstown, WV 25443
304-876-5106
www.shepherd.edu/cnsm

January 4, 2021

Dear Fellow Mathematics Educator,

The Shepherd University Mathematics department is continuing their annual mathematics contest, called the Shepherd Open. This will be an additional chance for high school students to compete, other than the already existing Math Field Day. We hope to instill a love of mathematics among high school-aged students by offering this contest. A special attraction is that there are cash prizes!

As with the Math Field Day, this is a grass roots effort, and we are asking for some help from you. We truly admire all the uncompensated volunteer work that many of our colleagues put into the Math Field Day. We are asking for much less here: as the contest is a take-home venture (students will have several weeks to work on the problems), no supervision is required.

All we are asking you to do is to advertise the contest for your students: talk about it in your classes, post the contest rules and problems in some way accessible to the students - like your school's website or Google classroom on or after January 6, 2021. All students are eligible to compete. The problems test originality and mathematical ingenuity rather than quantity of knowledge. They could be a good preparation for undergraduate contests like the Putnam and the VPI. We highly recommend a truly excellent book for preparing for this contest: Paul Zeitz: *The Art and Craft of Problem Solving*, 2nd ed. Wiley, 2006, ISBN 0471789011. There is also an excellent website with an enormous amount of top-grade mathematical content (mathematics contests, courses to prepare for these contests, recommended textbooks) - in short, a good cross section of the entire culture of mathematical problem solving is presented here: www.artofproblemsolving.com

Please see page two for official rules and page three for contest questions. If you have any questions about the Shepherd Open, please contact:

Dr. Nicholas Martin
304-876-5336
Email: horvat@shepherd.edu
Department Webpage: www.shepherd.edu/cme
Math Contest Web Address: www.shepherd.edu/cme/for-high-school-students

Thanks in advance,
Nick

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14th Annual Shepherd Open Mathematics Contest Rules

1. All public, private or home-schooled students are eligible to take part in this contest.

2. The duration of the contest is between January 6 – January 31, 2021. Students are allowed to consult textbooks and the internet, but are not allowed to get help from mathematicians or other persons.

3. Please enclose the following signed and dated statement: "I certify that all the work that I am submitting is original and I did not receive anyone's assistance

4. The solutions must be e-mailed by 11:59pm on January 31, 2021 to: horvat@shepherd.edu. Solutions may be typed or written by hand and photographed. Please make sure that the photo is legible and in .jpg format. Please include your name, your mathematics teacher's name(s) and your school name with your answers.

5. The first prize is $200 cash, the second prize is $150, third prize is $100 and an honorable mention would be considered for $50. The Shepherd Open reserves the right not to award any prizes if the quality of the solution does not meet our expectations. The winners will be invited to and recognized at an awards ceremony in April 2021, with the exact date to be determined.

6. The winners will be announced around the end of March 2021.

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14th Annual Shepherd Open Mathematics Contest Problems

I. Solve the system of equations:

$xy + yz = a$

$yz + zx = b$

$zx + xy = c$

Discuss necessary and sufficient conditions for the existence of solutions.

II . Point P is inside a circle of radius R, at distance d from the center. Chord AB is perpendicular to the diameter through P at P. Construct two squares, whose only point in common is their common vertex P, with their other vertices one each on the diameter and the chord, and the fourth vertex on the circle. Find the sum of their areas!

III. Consider a square of side *l* with vertices X, B, C, D. We cut off its vertex X, with a straight line, obtaining the pentagon with vertices A, B, C, D and E, with A on side XB, and E on side XD of the original square. Another (smaller) square is constructed so that two of its vertices are on side AE of the pentagon, one on BC and the other one on CD.
Let AB = a, DE = b. Find the side length of this smaller square. (a, b < *l*)